Interference phenomena in radiation of a charged particle moving in a system with one-dimensional randomness

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This paper considers the interference contribution in the radiation generated by a charged particle moving through a medium of randomly spaced parallel dielectric plates. For wavelengths much smaller than the photon mean free path, there appears in the angular dependent radiation intensity an ‘‘enhanced backscattering’’ peak in a cone with opening angle θ in the regime $\pi-\theta\sim\gamma^{-1}$, where γ is the Lorentz factor of the charged particle in the medium.

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I. INTRODUCTION

It is well known that a charged particle passing through a stack of randomly spaced dielectric plates radiates electromagnetic waves (see, for example, Ref. [1]). The radiation is caused by the scattering of the electromagnetic field (pseudophoton) of the charged particle from the inhomogeneities in the dielectric constant. In an earlier study one of us has shown [2] that, in analogy with three-dimensional random media [3], multiple scattering of the electromagnetic field plays an important role. In that work only the diffusion contribution was taken into account. At this level the approach is equivalent to the radiative transfer theory for light transport in, e.g., slab geometries; see Ref. [4] for a recent review. On the other hand, interference effects are important when waves propagate in random inhomogeneous media. Anderson localization [5] and the enhanced backscattering peak [6] are manifestations of these effects. Other interference effects show up in correlations and higher moments of the transmitted intensity. They were also reviewed in Ref. [4].

In the present paper we want to investigate interference effects for radiation of a charged particle moving in a system with one-dimensional randomness.

II. FORMULATION OF THE PROBLEM

The system which we want to study consists of a stack of plates randomly spaced in a homogeneous medium. It is convenient to represent the dielectric constant as the sum of an average and fluctuating part

$$\varepsilon(z,\omega)=\varepsilon+\varepsilon_r(z,\omega),\quad <\varepsilon_r(z,\omega)>=0. \tag{1}$$

Here $\langle ...\rangle$ means averaging over the random z coordinates of plates.

At an observation point $\mathbf{R}$ far away from the system ($R \gg r$) the radiation tensor is $I_{ij}(\mathbf{R})=E_{ri}(\mathbf{R})E^{*}_{rj}(\mathbf{R})$, where E_{ri} is the electric radiation field. This tensor consists of three contributions. The first two are the single scattering and the diffusion contributions. They have already been studied in a previous paper [2]. In the present paper we shall focus on the third one, the interference contribution.

For completeness we present the single scattering and interference contributions without derivation. It was shown in Ref. [2] that the single scattering contribution to the radiation intensity $I=(cR^2/2)I_{ii}(\mathbf{R})$ has the form

$$I^0(\theta)=\frac{e^2}{2c}\frac{L_zB(|k_0-k\cos\theta|)\sin^2\theta}{[\gamma^{-2}+\sin^2\theta k^2/k_0^2]^2}\frac{\omega^2}{k_0^4c^2}, \tag{2}$$

where $\gamma=(1-\varepsilon v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle in the medium. $\mathbf{n}$ is the unit vector in the direction of the observation point $\mathbf{R}$, having z component $n_z=\cos\theta$. Furthermore, $k_0=\omega/v$, $k=\omega\sqrt{\varepsilon}/c$, while L_z is the system size in the z direction. B is the correlation function of the dielectric constant, which is random in the z dimension

$$B(|z-z'|)=\frac{\omega^4}{c^4}<\varepsilon_r(z)\varepsilon_r(z')>. \tag{3}$$

It was shown in Ref. [2] that in our situation its Fourier transform reads

$$B(q_z)=\frac{\omega^4}{c^4}\frac{4(b-\varepsilon)^2n\sin^2q_za/2}{q_z^2}, \tag{4}$$

where n is the concentration of plates in the system, b is their dielectric constant, and a is their thickness. As seen from Eq. (2) at $ak\ll 1(B\sim\text{const})$, the forward and backward intensities are equal. When $ak\gg 1$, for relativistic particles $k_0\to k,\gamma\gg 1$ the forward intensity ($\theta\approx 0$) is significantly larger than the backward intensity ($\theta\approx\pi$) because of the factor B.

Next we present the diffusion contribution [2] to the radiation intensity

$$I^D(\omega,\theta)=\frac{5}{2}\frac{e^2\gamma^2}{\varepsilon c}\left(\frac{L_z}{l(\omega)}\right)^3\frac{\sin^2\theta}{|\cos\theta|}, \tag{5}$$

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where $l \sim 4k^2/B(0)$ is the photon mean free path in the z direction. Expression (5) is obtained in the weak scattering limit $\lambda \ll l \ll L$ and angles $kl|\cos\theta| \gg 1$ Note that there is a numerical inexactitude in Ref. [2]. A more precise approach would be to solve the appropriate Schwarzschild-Milne equation for the present problem. This could bring overall prefactors of order unity [8,4]. For our present purpose we shall not be interested in these effects.

It follows from Eqs. (2) and (5), that $I^D/I^0 \sim (L_z/l)^2 \gg 1$. This means that diffusion contribution is dominating the single scattering part. As one could expect, the forward and backward intensities in the diffusion contribution are equal to each other. Finally, we note the strong dependence of the diffusion contribution on the particle energy.

As was mentioned above, we have obtained expression (5) in the Cherenkov limit $k_0 \to k$. However, in order to separate the radiation caused by the fluctuations of dielectric constant from the Cherenkov one we believe that the condition $k_0 > k(v\sqrt{\varepsilon} < c)$ is always satisfied. In principle, within the plate an opposite condition $v\sqrt{b} > c$ can be satisfied. This is possible because usually the average dielectric constant ε is smaller than the dielectric constant of the plate b. In this case Cherenkov radiation can originate from each plate. However, the intensity of this radiation will be proportional to the total thickness of plates which is negligible compared to the system size in the z direction. Note that all our three contributions are proportional to the system size.

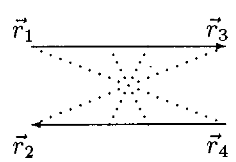
III. INTERFERENCE CONTRIBUTION

The interference contribution has the form

$$\begin{aligned} I_{ij}^C(\mathbf{R}) = \frac{k^2}{16\pi^2 R^2 \varepsilon} \int d\mathbf{r} d\mathbf{r}' B(r-r') A_0(\vec{r}) A_0^*(\mathbf{r}') \\ \times \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 \\ \times e^{-ik\mathbf{n}(\mathbf{r}_1-\mathbf{r}_2)} P^C(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) G(\mathbf{r}_3,\mathbf{r}) \\ \times G^*(\mathbf{r}',\mathbf{r}_4)[\delta_{\hat{z}i}\delta_{\hat{z}j} + n_i n_j n_z^2 - \delta_{\hat{z}i} n_j n_z - \delta_{\hat{z}j} n_i n_z], \end{aligned} \tag{6}$$

where P^C can be represented diagramatically as

$$P^C(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = \sum \tag{7}$$



The solid line denotes the averaged Green's function and the dotted one denotes the correlation function of the random dielectric constant. In the independent scatterer approximation the averaged Green's function has the form

$$G(\vec{q}) = \frac{1}{k^2 - q^2 + i\,\mathrm{Im}\,\Sigma(\mathbf{q})}, \tag{8}$$

where the imaginary part of the self-energy $\mathrm{Im}\,\Sigma(\mathbf{q})$ is determined self-consistently by the Ward identity

$$\begin{aligned} \mathrm{Im}\,\Sigma(\mathbf{q}) = \int \frac{d\mathbf{p}}{(2\pi)^3} B(\mathbf{p}) \mathrm{Im}\, G_0(\mathbf{q}-\mathbf{p}) = \frac{1}{4\sqrt{k^2-q_\rho^2}} \\ \times [B(|q_z - \sqrt{k^2-q_\rho^2}|) + B(|q_z + \sqrt{k^2-q_\rho^2}|)], \\ |\mathbf{q}_\rho| < k. \end{aligned} \tag{9}$$

Here $B(\mathbf{p}) = (2\pi)^2 \delta(\mathbf{p}_\rho) B(|p_z|)$, where $\mathbf{p}_\rho$ is the transverse component of $\mathbf{p}$. As mentioned above, the observation point is far away from the radiating system. For this reason, using Eq. (8), one can obtain the following useful relations for bare ($\mathrm{Im}\,\Sigma \to 0$) Green's function:

$$\begin{aligned} G_0(\mathbf{R},\mathbf{r}) \approx -\frac{1}{4\pi R} e^{ik(R-\mathbf{nr})}, \\ \frac{\partial^2 G_0(\mathbf{R},\mathbf{r})}{\partial R_i \partial_z} \approx \frac{k^2 n_i n_z}{4\pi R} e^{ik(R-\mathbf{nr})}, \quad R \gg r. \end{aligned} \tag{10}$$

We shall need relations (8)–(10) for the calculation of the interference contribution. The background potential appearing in Eq. (6) has the form [2]

$$A_0(\mathbf{q}) = -\frac{8\pi^2 e}{c} \frac{\delta(q_z - \omega/v)}{k^2 - q^2}. \tag{11}$$

As follows from Eq. (7), due to time-reversal invariance P^C is related to the diffusion propagator in the following manner (see, for example, Refs. [7,8]):

$$P^C(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = P(\mathbf{r}_1,\mathbf{r}_4,\mathbf{r}_3,\mathbf{r}_2). \tag{12}$$

Since $P(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$ is the sum of ladder diagrams [2], the diffusion propagator can be represented in the form

$$P(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = B(\mathbf{r}_1-\mathbf{r}_2) B(\mathbf{r}_3-\mathbf{r}_4) P(\mathbf{R}',\mathbf{r}_1-\mathbf{r}_2,\mathbf{r}_3-\mathbf{r}_4), \tag{13}$$

where $\mathbf{R}' = 1/2(\mathbf{r}_3+\mathbf{r}_4-\mathbf{r}_1-\mathbf{r}_2)$ and P has the limiting behavior

$$P(\mathbf{K}\to 0,\mathbf{p},\mathbf{q}) = \frac{\mathrm{Im}\, G(\mathbf{p}) \mathrm{Im} G(\mathbf{q})}{\mathrm{Im}\,\Sigma(\mathbf{q})} A(\mathbf{K}), \tag{14}$$

where

$$A(\mathbf{K}) = \frac{60\pi}{kl^2} \frac{1}{3K_z^2 + 2K_\rho^2}. \tag{15}$$

[Notice that in Eq. (41) of Ref. [2] there occurs a misprint in the numerical factors of $A(\mathbf{K})$.] In our previous paper [2] we investigated the special case $K_\rho = 0$, which was sufficient for studying the diffusion contribution. Using Eqs. (12) and (13) and changing the variables of integration by formulas

$$\mathbf{x}_1 = \mathbf{r}_1 - \mathbf{r}_4, \mathbf{x}_2 = \mathbf{r}_3 - \mathbf{r}_2, \mathbf{R}' = \frac{1}{2}(\mathbf{r}_3 + \mathbf{r}_2 - \mathbf{r}_1 - \mathbf{r}_4), \mathbf{r}_4 \equiv \mathbf{r}_4 \tag{16}$$

we find from Eq. (6)

$$
\begin{aligned}
I^C(\mathbf{n}) = {} & \frac{(1-n_z^2)ck^2}{32\pi^2\varepsilon} \\
& \times \int d\mathbf{r}d\mathbf{r}' B(r-r')A_0(\mathbf{r})A_0^*(\mathbf{r}') \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{R}' d\mathbf{r}_4 \\
& \times \exp\{ik\vec{n}\cdot[\vec{R}-(\vec{x}_1+\vec{x}_2)/2]\} \\
& \times P(\mathbf{R}',\mathbf{x}_1,\mathbf{x}_2)B(\mathbf{x}_1)B(\mathbf{x}_2) \\
& \times G\left(\mathbf{R}'+\frac{\mathbf{x}_1+\mathbf{x}_2}{2}+\mathbf{r}_4-\mathbf{r}\right)G^*(\mathbf{r}'-\mathbf{r}_4). \qquad (17)
\end{aligned}
$$

In the Fourier representation one finds from Eq. (17)

$$
\begin{aligned}
I^C(\mathbf{n}) = {} & \frac{(1-n_z^2)ck^2}{32\pi^2\varepsilon} \int \frac{d\mathbf{q}d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{K}}{(2\pi)^{12}} \\
& \times B(\mathbf{q})|A_0(-k\mathbf{n}-\mathbf{K}-\mathbf{q})|^2 B(\mathbf{q}_1)B(\mathbf{q}_2) \\
& \times P\left(\mathbf{K},\mathbf{k},\mathbf{n}+\frac{\mathbf{K}}{2}-\mathbf{q}_1,k\mathbf{n}+\frac{\mathbf{K}}{2}-\mathbf{q}_2\right)|G(k\mathbf{n}+\mathbf{K})|^2. \qquad (18)
\end{aligned}
$$

Substituting Eqs. (8), (11), and (14) into Eq. (18), taking into account that the main contribution in the integral over $\mathbf{K}$ come from the region $K\to 0$, and sequentially integrating Eq. (18) using the Ward identity (9), we find from Eq. (18)

$$
\begin{aligned}
I^C(\mathbf{n}) = {} & \frac{10\pi e^2 L_z |n_z|(1-n_z)^2 B(|k\cos\theta+k_0|)}{\varepsilon c l} \\
& \times \int \frac{d\mathbf{K}}{(2\pi)^3} \frac{1}{(3K_z^2+2K_\rho^2)[(\mathbf{K}_\rho+k\mathbf{n}_\rho)^2+k_0^2\gamma^{-2}]^2}. \qquad (19)
\end{aligned}
$$

Note that for $ak\gg 1$ one gets $B(2k)/B(0)\sim 1/k^2a^2\ll 1$. It follows from Eq. (19) in the Cherenkov limit $k\to k_0$ that the backward intensity $(\theta\approx\pi)$ is significantly larger than the forward intensity $(\theta\approx 0)$. This is the main characteristic feature of the interference contribution. It is analogous to the enhanced backscatter peak which occurs in propagation of light in the randomly inhomogeneous media [6]. However, there are essential differences. As is seen from Eq. (19), the angular width of the peak is of order $\lambda/2\pi a$, where a is the thickness of the plates, while in the case of diffuse light propagation it is of order λ/l, where l is the mean free path. Then the peak disappears in the "white noise" $ka\ll 1$ case. Considering angles $\sin\theta\gg\lambda/2\pi L_\rho$ and calculating the integral over K in Eq. (19) we have

$$
\begin{aligned}
I^C(\theta) = {} & \frac{5\sqrt{2}\arctan\sqrt{2}}{2\pi\varepsilon}\frac{e^2}{c}\frac{L_z}{l^2}B(|k\cos\theta+k_0|) \\
& \times \frac{\sin^2\theta|\cos\theta|}{[k^2\sin^2\theta+k_0^2\gamma^{-2}]^2}. \qquad (20)
\end{aligned}
$$

For obtaining Eq. (20) we cut off the integral over K at an upper limit equal to $1/l$. In the optical region the ratio $\lambda/2\pi L_\rho$ is of order $\sim 10^{-4}-10^{-5}$. Therefore the condition $\sin\theta\gg\lambda/2\pi L_\rho$ is always satisfied in the optical region. Comparing Eq. (20) with the single scattering contribution Eq. (2) we see that $I^C/I^0\sim 1/(kl)^2\ll 1$.

The interference contribution, though small, has quite different angular dependence. Accounting for the form of the correlation function B of Eq. (4), it follows from Eq. (20) that the maximum of the radiation intensity for relativistic particles having $\gamma\gg 1$, so $k_0\to k$, lies in the angular region $\theta\sim\pi-\gamma^{-1}$. These angles are very close to the backward direction. On the other hand, the maximum of the single scattering contribution lies in the strongly forward range of small angles, $\theta\sim\gamma^{-1}$.

So, by investigating the angular dependence of the radiation intensity for the angles close to the peak one $\pi-\theta\sim\gamma^{-1}$, it is possible to pick out the interference contribution, because the diffusion and single scattering contributions do not have any peculiarities at these angles.

IV. SUMMARY

We have considered the influence of interference effects on the radiation of a charged particle passing through a stack of randomly spaced dielectric plates. It appears that the interference contribution to the radiation intensity has a peak in the backward to particle motion direction. Though its value is small compared to the single scattering and diffusion contributions, it can be investigated experimentally. This is possible due its specific angular dependence.

[1] M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (Wiley, New York, 1972).
[2] Zh. S. Gevorkian, Phys. Rev. E **57**, 2338 (1998).
[3] Zh. S. Gevorkian, Phys. Lett. A **162**, 187 (1992); Radiofizika **36**, 36 (1993).
[4] M. C. W. van Rossum and Th. M. Nieuwenhuizen, Rev. Mod. Phys. **71**, 313 (1999); e-print cond-mat/9804141.
[5] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[6] M. P. van Albada and A. Lagendijk, Phys. Rev. Lett. **55**, 2692 (1985).
[7] B. L. Altshuler, A. G. Aronov, D. E. Khmelnitskii, and A. I. Larkin, in *Quantum Theory of Solids*, edited by I. M. Lifshits (Mir, Moscow, 1982), p. 130.
[8] Th. M. Nieuwenhuizen and J. M. Luck, Phys. Rev. E **48**, 569 (1993).